Christ Church Grammar School

Mathematics Department
Year 11 Mathematics Methods

Semester 1 Examination, 2019

Question/Answer Booklet

MATHEMATICS METHODS

UNIT 1

Section One: Calculator Free

Fix student label here

Student Name: _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules of conduct of the CCGS assessments are detailed in the Reporting and Assessment Policy. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section One: Calculator-free **35% (52 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(5 marks)**

Solve the following equations for x.

(a) $(5x-3)(x+4)=0$. (1 mark)

(b) $\frac{x}{2}=\frac{4x-3}{3}$. (2 marks)

(c) $2x^2=6x$. (2 marks)

See next page

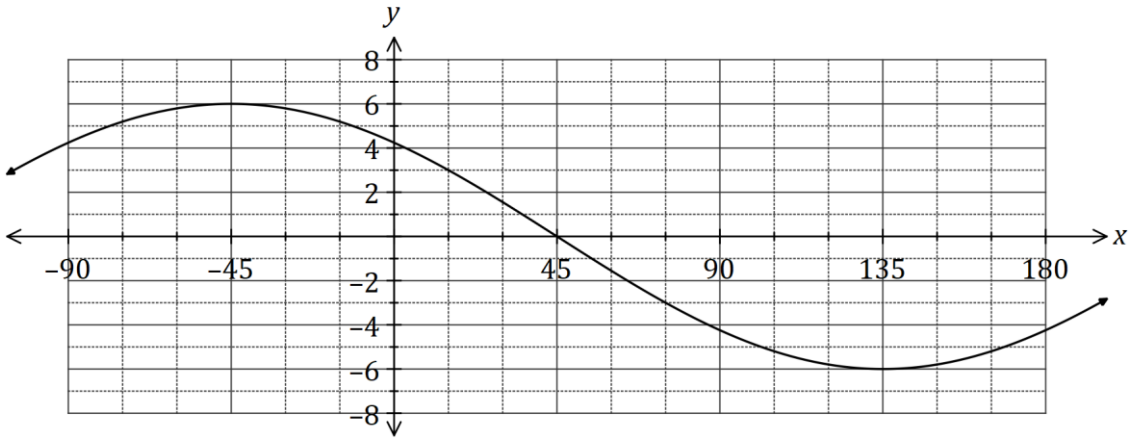
Question 2 **(5 marks)**

(a) A circle of radius 2 has its centre at the point $(1, -4)$. Determine the equation of the circle in the form $x^2 + y^2 = ax + by + c$. (3 marks)

(b) The graph of $x = y^2$ passes through the point $(9, q)$. Determine the value(s) of q and hence explain why y is a relation but not a function of x. (2 marks)

Question 3 **(6 marks)**

(a) The graph of $y = a\cos(x + b)$ is shown below, where a and b are constants.



Determine the value of a and the value of b, where $-90° \leq b \leq 180°$. (2 marks)

(b) Given that $0° \leq x \leq 360°$, solve

(i) $\cos(x) = \frac{1}{2}$. (1 mark)

(ii) $8\cos(x + 30°) + 4\sqrt{3} = 0$. (3 marks)

See next page

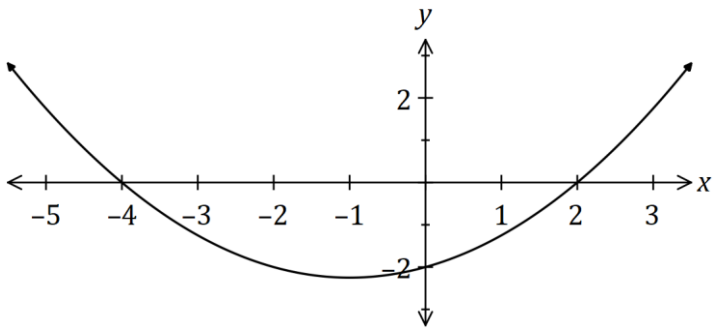
Question 4 **(7 marks)**

(a) Determine the coordinates of the

(i) y-intercept of the graph of $y = -2(x + 4)^2 + 12$. (1 mark)

(ii) turning point of the graph of $y = (x - 3)(x + 1)$. (2 marks)

(b) The graph of $y = ax^2 + bx + c$ is shown below. Determine the value of the coefficients a, b and c. (4 marks)



Question 5 **(7 marks)**

(a) Expand $x(x+5)^2$. (2 marks)

(b) Let $f(x) = x^3 + 2x^2 - 11x - 12$.

(i) Determine $f(-1)$. (1 mark)

(ii) Solve $f(x) = 0$. (4 marks)

See next page

Question 6 **(7 marks)**

(a) Describe the behaviour of the y values for each of the following graphs, given the behaviour of the x values:

(i) $y = x^4$, as $x \to \infty$. (1 mark)

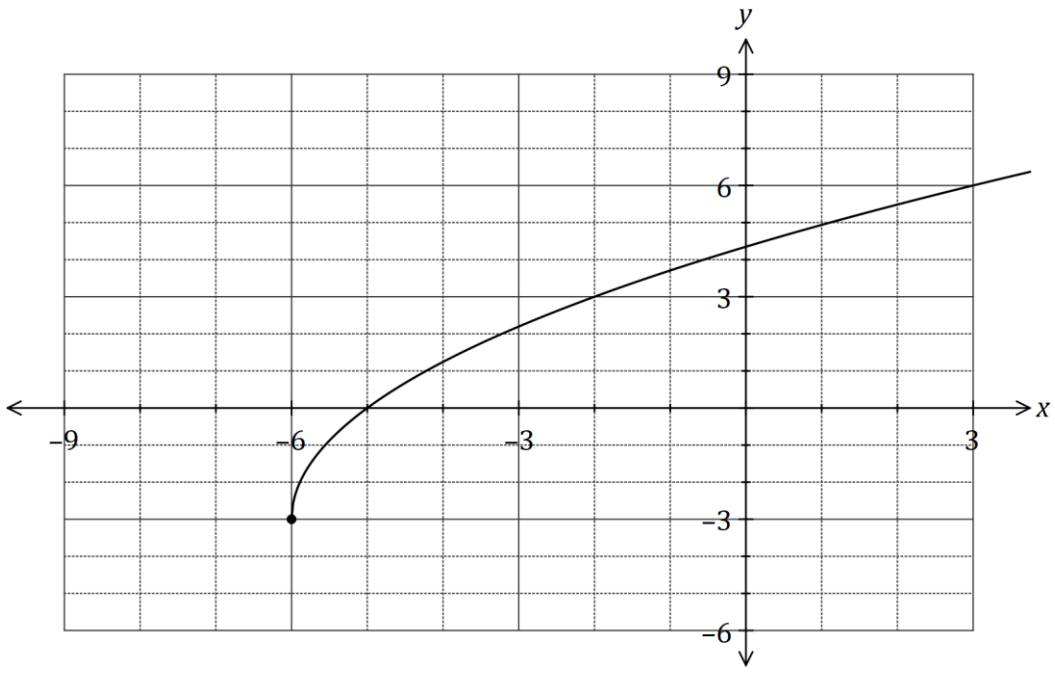
(ii) $y = (2 - x)^3$, as $x \to \infty$. (1 mark)

(iii) $y = \frac{1}{x}$, as $x \to -\infty$. (1 mark)

(b) The graph of $y = f(x)$ is shown below. On the same axes sketch the graph of

(i) $y = f(x + 3)$. (2 marks)

(ii) $y = f(3x)$. (2 marks)



Question 7 **(8 marks)**

(a) Complete the row of Pascal's triangle that starts $1, 5, 10, \ldots$ and express the sum of the numbers in this row as a power of 2. (2 marks)

(b) Use ${}^{n}C_{r}$ notation to write down the seventh number in the row of Pascal's triangle that starts with 1, 8, 28, … (1 mark)

(c) Determine the coefficient of

(i) the x^2 term in the expansion of $(7x - 2)^2$. (1 mark)

(ii) the x^4 term in the expansion of $(x + 1)^5$. (1 mark)

(iii) the x^3 term in the expansion of $(2 - 3x)^5$. (3 marks)

See next page

Question 8 **(7 marks)**

(a) Evaluate $\sin\left(\frac{39\pi}{4}\right)$. (2 marks)

(b) A is an acute angle and B is an obtuse angle such that $\cos A = \frac{1}{3}$ and $\sin B = \frac{2}{3}$.

(i) Show that $\sin A = \frac{2\sqrt{2}}{3}$ and determine the value of $\cos B$. (3 marks)

(ii) Determine the value of $\sin(A + B)$ as a single fraction. (2 marks)

End of questions

Christ Church Grammar School

Mathematics Department
Year 11 Mathematics Methods

Semester 1 Examination, 2019

Question/Answer Booklet

MATHEMATICS METHODS

UNIT 1

Section Two: Calculator Assumed

Fix student label here

Student Name: _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

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Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	14	14	100	98	65
				Total	100

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See next page

Section Two: Calculator-assumed **65% (98 Marks)**

This section has **fourteen (14)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(4 marks)**

(a) The points A and B have coordinates $(7, -2)$ and $(-3, 6)$ respectively. If A is the midpoint of B and C, determine the coordinates of C. (2 marks)

(b) The points D and E have coordinates $(-2p, q)$ and $(3q, -2p)$ respectively, where p and q are constants. Determine the value of p and the value of q if the midpoint of D and E is at $(11, -7)$. (2 marks)

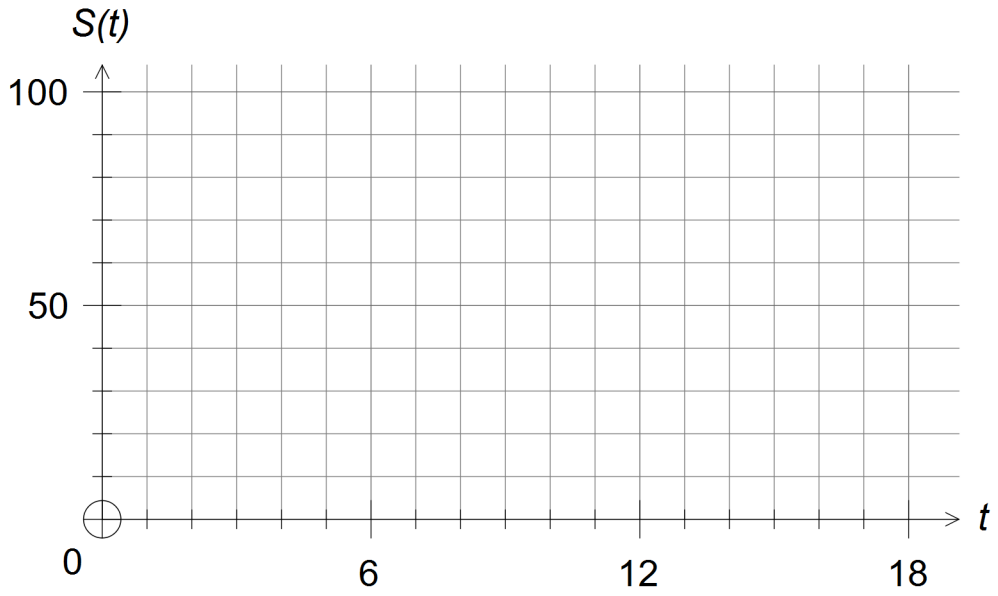
See next page

Question 10 **(8 marks)**

The average wind speed, $S(t)$ in km/h, over an 18 hour period from midnight to 6pm during a stormy day was observed to follow $S(t) = \frac{t^3}{10} - \frac{5t^2}{2} + 16t + 28$ where t was the number of hours since midnight.

(a) No data was available after 6pm as the measuring instrument broke at that time. What was the average wind speed at 6pm? (1 mark)

(b) On the grid below, sketch a graph to show how the average wind speed varied during the 18 hour period. (2 marks)



(c) At the height of the storm in the **morning**, some properties suffered structural and other damage. At what time, to the nearest quarter of an hour, did this occur? (2 marks)

Question 10 continued

(d) What was the lowest average wind speed recorded after 6am? (1 mark)

(e) For what percentage of the 18 hours did the average wind speed exceed 50 km/h? (2 marks)

See next page

Question 11 **(7 marks)**

Line L has equation $\frac{x}{5}+\frac{y}{3}=1$.

(a) State the coordinates of the point where L intersects the x-axis. (1 mark)

(b) State, with justification, if L is parallel to the line with equation $y=0.6x+4$. (2 marks)

(c) Determine the equation of line P that is perpendicular to L and passes through the point with coordinates $(30, 19)$. (2 marks)

(d) Determine the coordinates of the point of intersection of L and P. (2 marks)

Question 12 **(8 marks)**

(a) The variables Q and v are directly proportional and when $v = 40, Q = 10$.

(i) Determine an equation for the relationship between Q and v. (2 marks)

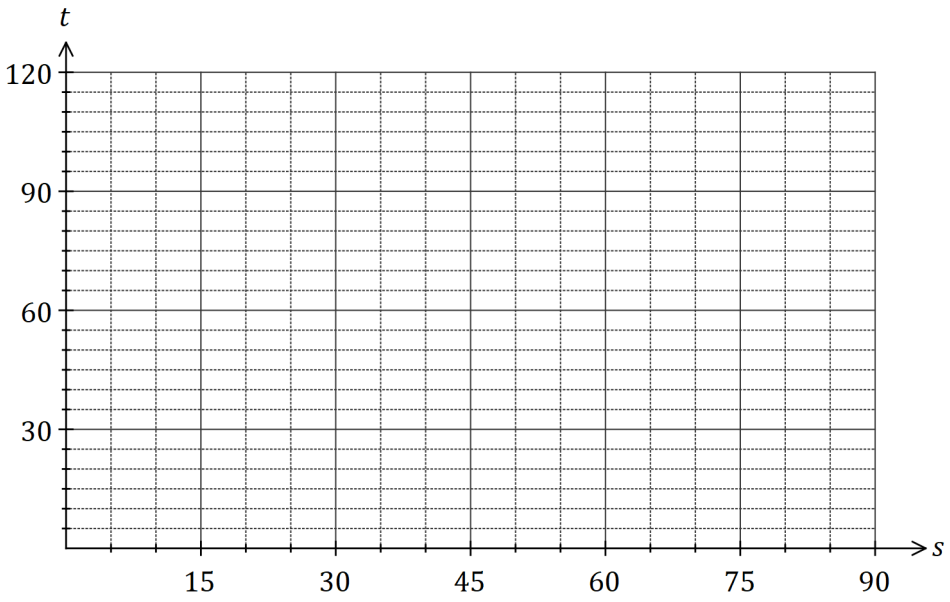
(ii) State the value of Q when $v = 80$. (1 mark)

(b) The time, t minutes, that a car takes to travel 250 m at a constant speed of s kmh^{-1} is given by the formula $t = \frac{k}{s}$.

(i) Determine the value of the constant k, given that when $s = 15, t = 60$. (1 mark)

(ii) Determine the value of t when $s = 10$. (1 mark)

(iii) On the axes below, draw a graph to show how t varies with s. (3 marks)



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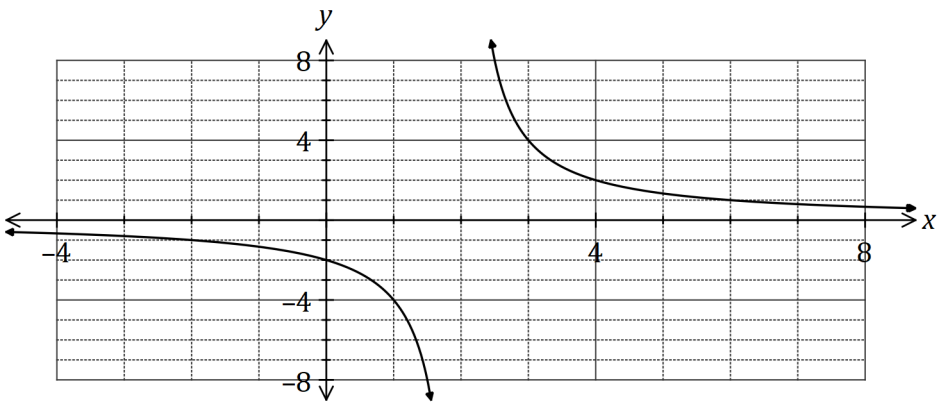
Question 13 **(8 marks)**

The graph of $y = f(x)$ is shown below where $f(x) = \frac{a}{b - x}$.



(a) State the value of the constant a and the value of the constant b. (3 marks)

(b) The hyperbola shown above has two asymptotes. State their equations. (2 marks)

(c) Describe how to transform the graph of $y = f(x)$ to obtain the graph of $y = f(x + 1)$ and state the domain and range of the transformed function. (3 marks)

Question 14 **(8 marks)**

(a) Convert, giving an exact answer

(i) $40°$ to radians. (1 mark)

(ii) 0.2 radians to degrees. (1 mark)

(b) Calculate, to the nearest degree, the acute angle between the line $y = 4.5x + 2$ and the line $y = 1.5x - 3$. (3 marks)

(c) The sides adjacent to the right-angle in a right triangle have lengths 36 cm and 77 cm.

If the smallest angle in the triangle is α, then determine an exact value for

(i) $\tan \alpha$. (1 mark)

(ii) $\cos(90° - \alpha)$. (2 marks)

See next page

Question 15 **(7 marks)**

An **obtuse** angled triangle WXY has $w = 45$ cm, $y = 34$ cm and an area of 739 cm^2.

(a) Sketch a triangle to show this information. (1 mark)

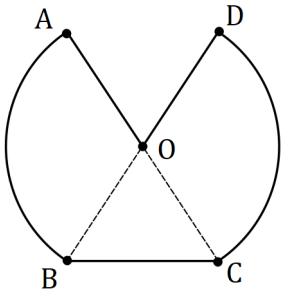
(b) Determine the size of $\angle X$. (2 marks)

(c) Show that $x \approx 63$ cm. (2 marks)

(d) Show that $\angle Y \approx 31°$. (2 marks)

Question 16 **(7 marks)**

In shape $OABCD$ below, $\angle AOB = 117°$ and AC, BD are diameters of the circle with centre O and radius 42 cm.



(a) Calculate the perimeter of $OABCD$. (4 marks)

(b) Calculate the area of $OABCD$. (3 marks)

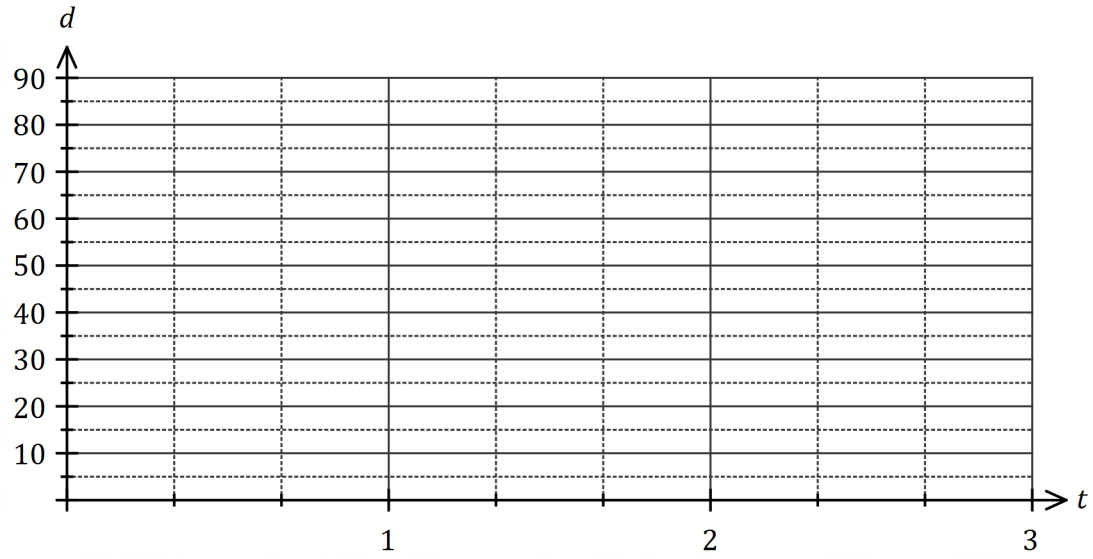
See next page

Question 17 **(7 marks)**

A small mass, attached to the bottom of a spring, oscillated up and down. The distance, d cm, of the mass from the top of the spring after t seconds can be modelled by

$$d = 45 + 35\sin\left(\frac{3\pi t}{4}\right)$$

(a) Sketch the graph of d against t on the axes below for $0 \leq t \leq 3$. (3 marks)



(b) Mark on your graph point M, where the mass is 40 cm from the top of the spring and moving downwards. (1 mark)

(c) Determine

(i) the maximum distance of the mass from the top of the spring. (1 mark)

(ii) the time taken for the mass to first return to its initial position. (1 mark)

(iii) the distance moved by the weight between $t = 1$ and $t = 2$. (1 mark)

See next page

Question 18 **(8 marks)**

(a) The equation of the axis of symmetry for the graph of $y = 2x^2 + 8x + 5$ is $x = m$. Determine the value of m, using a method that does not refer to the graph of the parabola.
(2 marks)

(b) A parabola with equation $y = ax^2 + bx + c$ has a turning point at $(4, -5)$ and passes through the point $(2, -17)$. Determine the value of a, the value of b and the value of c.
(3 marks)

(c) Determine the value of the discriminant for the quadratic equation $4x^2 - 28x + 47 = 0$ and use it to explain how many solutions the equation $(x + 3)(4x^2 - 28x + 47) = 0$ will have.
(3 marks)

See next page

Question 19 **(6 marks)**

Let $p = \cos 130°$ and $q = \sin 35°$.

Give your answers to the following in terms of p and/or q.

(a) Write down an expression for

(i) $\sin 145°$. (1 mark)

(ii) $\cos 50°$. (1 mark)

(b) Determine an expression for $\cos 145°$. (3 marks)

(c) Hence, determine an expression for $\tan 145°$. (1 mark)

Question 20 **(9 marks)**

(a) Without evaluating, show that: (3 marks)

$$\cos 15° \cos 65° + \sin 15° \sin 65° = \sin 140°$$

(b) Simplify $\sin(A + B) \cos B - \cos(A + B) \sin(B)$. (2 marks)

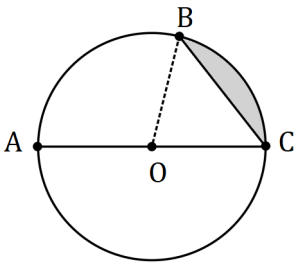
(c) (i) Show that $\sqrt{2} \sin(x + 45°) = \sin x + \cos x$ (2 marks)

(ii) Hence, show that the exact value of $\sin 75° = \frac{(1+\sqrt{3})}{2\sqrt{2}}$ (2 marks)

See next page

Question 21 **(8 marks)**

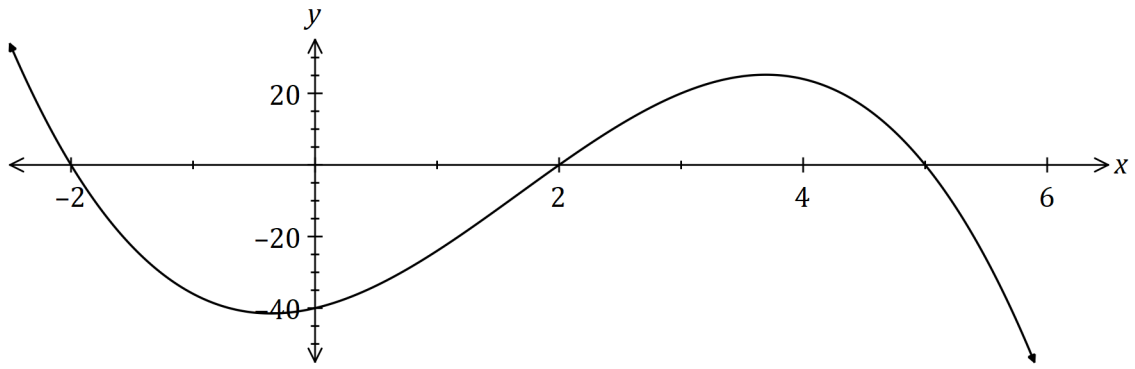
(a) The circle shown has centre O and diameter AC of length 50 cm. Determine the shaded area given that $2 \times \angle AOB = 3 \times \angle BOC$. (4 marks)



(b) A sector of a circle has a perimeter of 112 cm and an area of 735 cm^2. Determine all possible values for the radius of the circle. (4 marks)

Question 22 **(3 marks)**

The graph of the cubic function $y = f(x)$ is shown below. Determine $f(10)$.



End of questions